# Kinetic Theory with Algebra

Suppose that the box contains $N$ molecules ($N$ in the whole box, not $N$ molecules in each cubic meter as in some texts). Suppose that the box has length $a$ meters and ends of dimensions $b$ meters by $c$ meters.

In the course of their random motion with many collision the molecules will exchange momentum, $p$, and will not all keep the same velocity. However, if the temperature is kept constant, we believe their velocities will range around a fixed average velocity, which we call $v$ meters/sec. To calculate the pressure on one end of the box we deal only with molecular impacts on that end. So to simplify the problem we pretend that the $N$ molecules are regimented in three equal groups, one lot moving up-and-down, one lot to-and-fro across the width, and one lot moving forwards-and-backwards along the length. For the pressure on one end we then consider the last lot only. Symmetry-considerations suggest we should imagine the molecules equally divided among the three groups. Making these assumptions, answer the questions below, using $m$ kilograms for the mass of one molecule.

(i) When one molecule hits the front end head-on and rebounds, its change of momentum is ....................................$2mv$.

(ii) Between successive impacts on the *front end* a molecule travels to the other end and back: a total distance $2d$ meters.

(iii) In a total time $t$ seconds, a molecule moving with velocity $v$ meters/sec travels a total distance ...................................$vt$.

(iv) $\therefore$ in $t$ seconds, a molecule can make $vt/2d$ round trips and so can make this number of impacts on the front end.

(v) $\therefore$ in $t$ seconds, a molecule makes $vt/2d$ impacts on the front end of box, suffering at each impact a change of momentum

$2mv$

(vi) $\therefore$ total change in momentum, due to impacts of *one* molecule, suffered by front end in $t$ seconds is ..........................$mv^2t/d$.

(vii) But there are $N$ molecules in the box, of which $1/3$ are in the group moving forward and backward between the ends.
$\therefore$ the total change of momentum, due to impacts of all molecules concerned, suffered by the front end in $t$ seconds is $Nmv^2t/3d$

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(viii) But, $Ft = \Delta p$ $\therefore F = \Delta p/t$
and in this case the average *Force*, during their period of $t$ seconds, on the front end of the box is[1] ...................... $Nmv^2/3d$

(ix) $P = F/A$, and the area of the end face is ................. $bc$.
$\therefore$ average *Pressure* on the end of the box is ......... $\frac{Nmv^2}{3bcd}$.

(x) The volume of the box is ............................... $bcd$ m$^3$.
$\therefore$ the product .................................. $PV = \frac{1}{3}Nmv^2$.
But $m$ is the mass of one molecule, and there are $N$ molecules, so the *total mass of gas* in the box $M$ kg = $Nm$ kg.
Substituting $M$ into the algebra above, we have

$$PV = \frac{1}{3}Mv^2$$

(xi) Providing we use a closed box or other apparatus allowing no leakage of gas, then $M$ is constant. Suppose we keep temperature constant; then other experiments in physics suggest that the average velocity $v$ remains constant. Then in this case when the volume is changed the result of (x) above suggests that $P$ changes inversely

(xii) If we measure the *volume* of a sample of gas, say in a globe, and find its mass (by weighing the globe full of gas and then evacuated), and measure the *pressure* of the sample with a barometer, then the result of (x) above enables us to calculate a very important piece of information, the value of $v$, which is the average velocity of the molecules.

(xiii) UNITS TO BE USED. In making the calculation of (xii) above if the volume is in m$^3$ the mass should be in ..........kilograms
and the pressure should be in .........................Pascals

(xiv) We have already derived two useful things from our molecular theory, a behavior-suggestion in (xi) and a very interesting measurement in (xii), and more results will emerge; but we must pay for them by the assumptions that go into the machine. *List on a separate sheet as many assumptions as you can*, (a) of general physical laws assumed to apply to molecules, Conservation of mass & energy & momentum / Newton's Equations / (b) of special properties, of behavior, size, etc., assumed for molecules. They don't collide / They don't take up any finite volume / No external potential /

[1] Here, $t$ is the time during which the average force would have to act to produce this momentum-change. Therefore $t$ IS the time $t$ seconds for which we have calculated the total momentum-change.