Name: ______________________________ §#:

Show all work clearly and in order, and box your final answers. Simplify your expressions as best you can. Use the back of the sheet if you need to. You have 10 minutes to take this quiz.

CONCENTRIC SPHERES - There exists an insulating sphere with radius R and uniform charge density, with total charge Q. Surrounding this sphere is conducting, spherical shell with inner radius $2R$ and outer radius $3R$. Surrounding this is a conducting spherical shell with inner radius $4R$ and outer radius $5R$, on which I have placed a total charge $-Q$. Lets agree to put 0 potential at infinity.

So, I think this quiz is easiest to answer by first calculating the electric field everywhere and then integrating to obtain the potential, i.e. using the relationship

$$V = -\int_a^b \vec{E} \cdot d\vec{l}$$

In particular, the electric field in this problem is easy to find in all regions with straight forward applications of Gauss's law, we find

$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\hat{r} \begin{cases} \frac{r}{R^3} & r < R \\ \frac{1}{r^2} & R < r < 2R \\ 0 & 2R < r < 3R \\ \frac{1}{r^2} & 3R < r < 4R \\ 0 & 4R < r < 5R \\ 0 & r > 5R \end{cases}$$

1. (*4 points*) Sketch the potential as a function of r

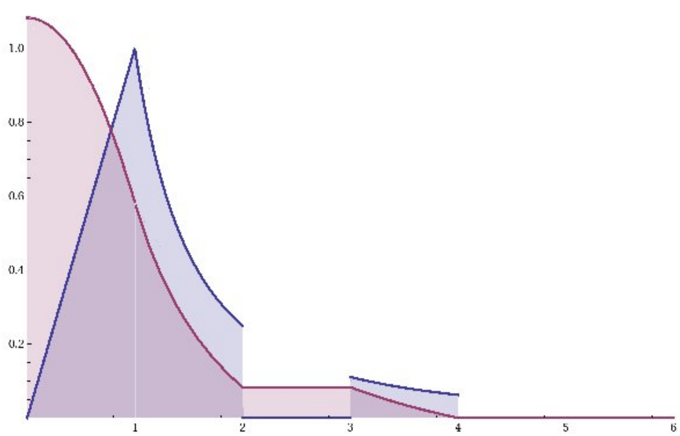


Figure 1: Plot of the Electric field (blue) and corresponding potential (purple) over the range of interest. The x axis is in units of R and the y axis in units of $\frac{Q}{4\pi\epsilon_0}$

2. (*4 points*) What is the electric potential everywhere?

The potential everywhere is just the integral of the field above, since we agreed to put the zero point of the potential at infinity in particular, the potential is given by

$$V(r) = -\int_{\infty}^{r} \vec{E}(r)\cdot d\vec{l} = \int_{r}^{\infty} E(r)\ dr$$

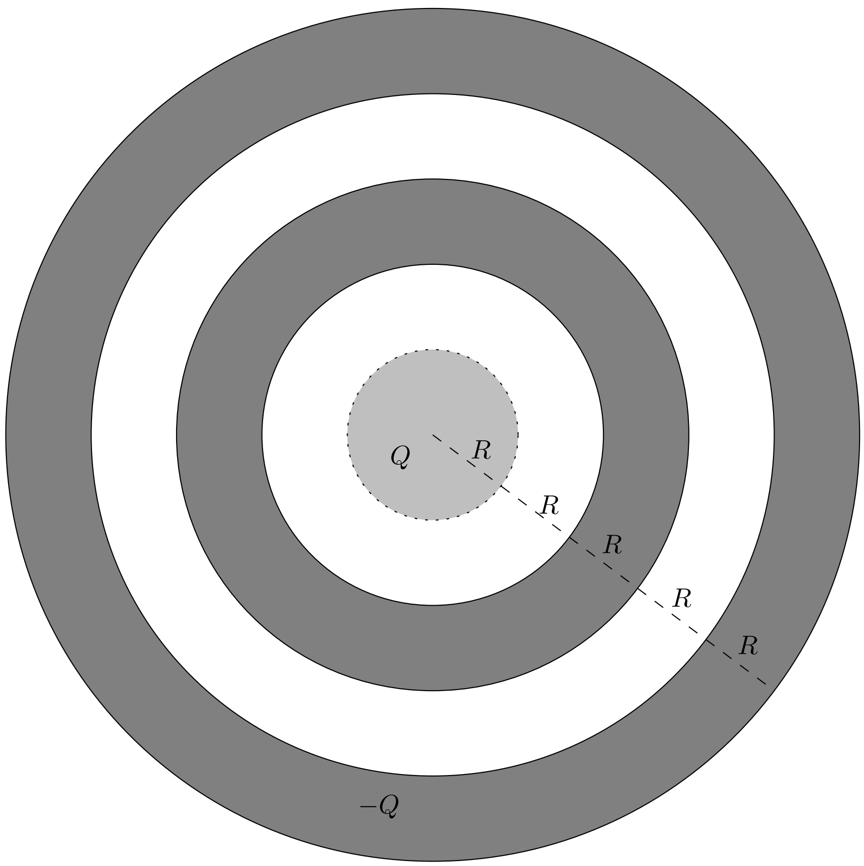
or

$$V(r) = \frac{Q}{4\pi\epsilon_0}\begin{cases} \frac{R^2}{2R^3} - \frac{r^2}{2R^3} + \frac{1}{R} - \frac{1}{2R} + \frac{1}{3R} - \frac{1}{4R} & r < R \\ \frac{1}{r} - \frac{1}{2R} + \frac{1}{3R} - \frac{1}{4R} & R < r < 2R \\ \frac{1}{3R} - \frac{1}{4R} & 2R < r < 3R \\ \frac{1}{r} - \frac{1}{4R} & 3R < r < 4R \\ 0 & 4R < r < 5R \\ 0 & r > 5R \end{cases}$$

which simplifies quite a bit to

$$V(r) = \frac{Q}{4\pi\epsilon_0}\begin{cases} \frac{13}{12R} - \frac{r^2}{2R^3} & r < R \\ \frac{1}{r} - \frac{5}{12R} & R < r < 2R \\ \frac{1}{12R} & 2R < r < 3R \\ \frac{1}{r} - \frac{1}{4R} & 3R < r < 4R \\ 0 & 4R < r < 5R \\ 0 & r > 5R \end{cases}$$

3. (*2 points*) At what values of r are there discontinuities in the potential? Why?
There are no discontinuities in the potential, and we should never expect to find any discontinuities in the potential, for if there where that would correspond to an infinite electric field which cannot exist. Thinking about it another way, at worst the electric field can be piecewise discontinuous, and the integral of a piecewise discontinuous function is always continuous.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