Name: _____ §#:

Show all work clearly and in order, and box your final answers. Simplify your expressions as best you can. Use the back of the sheet if you need to. You have 10 minutes to take this quiz.

CAPACITOR NETWORK - Consider the capacitor network drawn on the board. Answer the following questions. Initially, between the points a, b, I have connected a 10 V power supply. It has been connected for quite some time.

So before we get into it, lets note that we know that for capacitors in series, their charges have to be the same, and for capacitors in parallel their voltages have to be equal. Furthermore, lets call the equivalent capacitance of the three in parallel on the left $C_L = C_A + C_B + C_C$ since they are in parallel. At this point we know that the charges on all three are the same, i.e. $Q_L = Q_D = Q_E$. And this voltage equals the voltage across all three, i.e. $V_L = V_A = V_B = V_C$, and that the charge on the equivalent capacitor is the sum of the charges on all three, i.e. $Q_L = Q_A + Q_B + Q_C$. Finally we can calculate these charges by using Kerchoff's Law for the whole lot. We have that the sum of the voltages must be the supplied voltages

$$V = \frac{Q_L}{C_L} + \frac{Q_D}{C_D} + \frac{Q_E}{C_E} = Q_L\left(\frac{1}{C_L} + \frac{1}{C_D} + \frac{1}{C_E}\right)$$

which gives

$$Q_L = Q_D = Q_E = \frac{V}{\frac{1}{C_L} + \frac{1}{C_D} + \frac{1}{C_E}}$$

1. (*1 point*) What is $\frac{Q_B}{Q_D}$?

So here we can compute

$$\frac{Q_B}{Q_D} = \frac{C_B V_B}{Q_D} = \frac{C_B V_L}{Q_D} = \frac{C_B \frac{Q_L}{C_L}}{Q_D} = \frac{C_B}{C_L}$$

So we've found

$$\boxed{\frac{Q_B}{Q_D} = \frac{C_B}{C_L} = \frac{C_B}{C_A + C_B + C_C}}$$

2. (*1 point*) What is $\frac{Q_A}{Q_B}$?

So here we know $Q_A = C_A V_A$ and $Q_B = C_B V_B$ but we know $V_A = V_B$ so the ratio just becomes

$$\boxed{\frac{Q_A}{Q_B} = \frac{C_A}{C_B}}$$

3. (*1 point*) What is $\frac{Q_B}{Q_C}$?

Following the same logic as the part above we have

$$\boxed{\frac{Q_B}{Q_C} = \frac{C_B}{C_C}}$$

4. (*1 point*) What is $\frac{V_A}{V_C}$?
From here we know that since these capacitors are in parallel they have the same voltage we have

$$\boxed{\frac{V_A}{V_C} = 1}$$

5. (*1 point*) What is $\frac{V_D}{V_E}$?
So here we have

$$\frac{V_D}{V_E} = \frac{Q_D/C_D}{Q_E/C_E} = \frac{C_E}{C_D}$$

So we've found

$$\boxed{\frac{V_D}{V_E} = \frac{C_E}{C_D}}$$

6. (*1 point*) What is $\frac{V_B}{V_E}$?
Here we have

$$\frac{V_B}{V_E} = \frac{V_L}{Q_E/C_E} = \frac{Q_L/C_L}{Q_E/C_E} = \frac{C_E}{C_L}$$

So we've found

$$\boxed{\frac{V_B}{V_E} = \frac{C_E}{C_A + C_B + C_C}}$$

Now I disconnect the power supply from the points a, b.

7. (*1 point*) What is V_{ab}?
So if they were connected a long time, the capacitor network should charge up to the point where their voltage is the same as the original voltage, i.e. $\boxed{V_{ab} = V_{bat}}$
Now I connect a 5 $k\Omega$ Resistor across a and b.

8. (*2 points*) What happens?
The capacitors will begin to discharge through the resistor. The charges will start flowing, and will continue flowing until the capacitors are all uncharged. In fact, the current should follow a decaying exponential.

9. (*1 point*) What is the time constant for this new circuit?
The time constant for an RC circuit is $\tau = RC$, so we need the know the total equivalent capacitance, which is given by

$$C_{tot} = \frac{1}{\frac{1}{C_L} + \frac{1}{C_D} + \frac{1}{C_E}}$$

and our time constant is

$$\boxed{\tau = RC_{tot}}$$