Name: _____ §#:

Show all work clearly and in order, and box your final answers. Simplify your expressions as best you can. Use the back of the sheet if you need to. You have 10 minutes to take this quiz.

CONCENTRIC SPHERES - There exists an insulating sphere with radius R and uniform charge density ρ. Surrounding this sphere is conducting, grounded spherical shell with inner radius $2R$ and outer radius $3R$. Surrounding this is a conducting spherical shell with inner radius $4R$ and outer radius $5R$, on which I have placed a total charge Q.

1. (*2 points*) What is the electric field in the region $R \in (0, R)$?

We've done this problem a number of times. The electric field inside an insulator with uniform charge density is linear in the radius. It takes the form

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}\left(\frac{r}{R}\right)\hat{r}$$

which in our case, $Q = \rho\frac{4}{3}\pi R^3$ we obtain

$$\boxed{\vec{E} = \frac{1}{3}\frac{\rho}{\epsilon_0} r\hat{r}}$$

if you didn't remember that it should grow linearly, we can obtain the same result using Gauss's Law

$$\int \vec{E}\cdot d\vec{A} = E(r)4\pi r^2 = Q_{\text{enclosed}}\epsilon_0 = \frac{4}{3}\pi r^3\frac{\rho}{\epsilon_0}$$

which gives us the same result

$$\boxed{\vec{E} = \frac{1}{3}\frac{\rho}{\epsilon_0} r\hat{r}}$$

2. (*1 point*) What is the electric field in the region $R \in (R, 2R)$?

In this region, we can treat the inner insulator as though it was a point charge with total charge $Q = \frac{4}{3}\pi R^3\rho$ located at the center, and we have the field due to a point charge

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

so

$$\boxed{\vec{E} = \frac{1}{3}\frac{\rho}{\epsilon_0}\frac{R^3}{r^2}\hat{r}}$$

3. (*1 point*) What is the electric field in the region $R \in (2R, 3R)$?

$$\boxed{\vec{E} = 0}$$

Since we are inside a conductor

4. (*1 point*) What is the electric field in the region $R \in (3R, 4R)$?

$$\boxed{\vec{E} = 0}$$

Since our grounded conductor shields the inner insulator.

5. (*1 point*) What is the electric field in the region $R \in (4R, 5R)$?

$$\boxed{\vec{E} = 0}$$

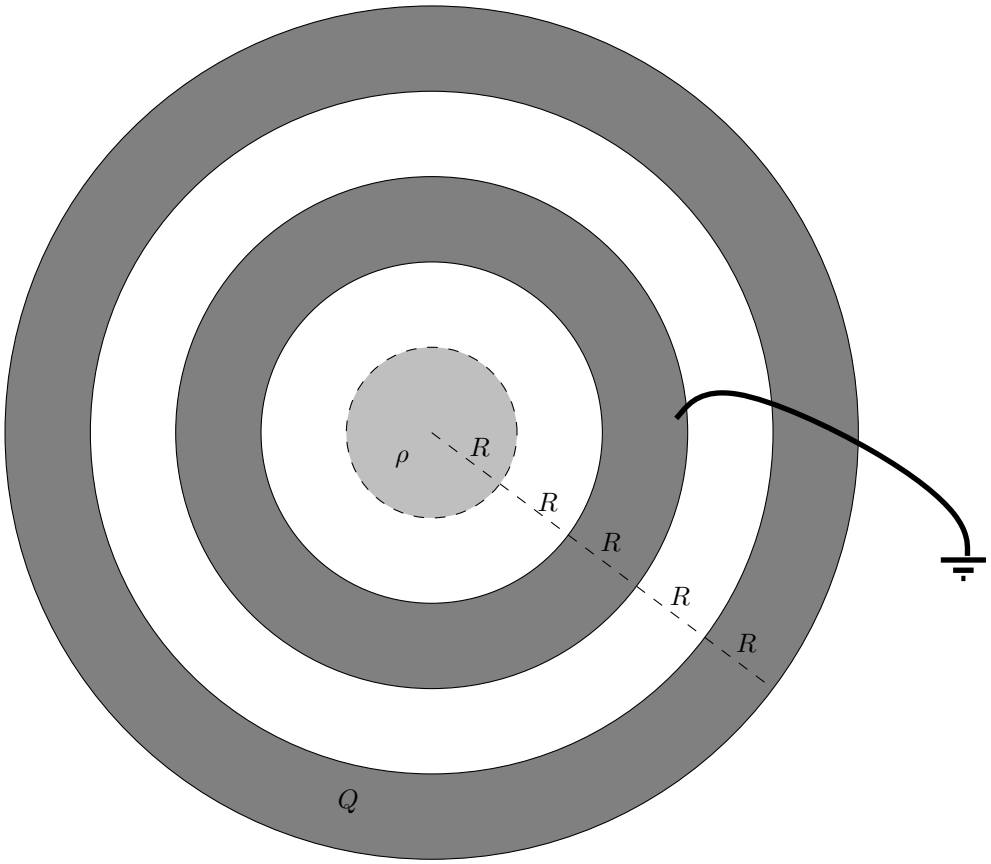
Since we are inside a conductor

6. (*2 points*) What is the electric field for $R > 5R$?
Since our grounded conductor sheilds the insulator, the only charge we need to worry about is the charge Q on the outer conductor, which will all be located at the outer surface. Again we can treat the field outside though as the field due to a point charge located at the center, we obtain

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

7. (*2 points*) At what values of R are there surface charge densities?
Surface charge densities cause discontinuities in our field. We expecte surface charges to be present anywhere our field had a discontinuity. So, these occur at

$$\boxed{R = \{2R, 5R\}}$$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