2213 - Heat and E&M - 2009
Quiz - 10/07/09 Name: ______________________________ §#:

Show all work clearly and in order, and box your final answers. Simplify your expressions as best you can. Use the back of the sheet if you need to. You have 10 minutes to take this quiz.

CHARGED PLATES - There exists two uniformly charged insulating plates or infinite extent. One has a constant volume charge density $\rho > 0$ the other, $-\rho$. They are separated by some distance d, and each of width w. A side view diagram can be found on the back side of this sheet. Be sure to explain any coordinates you introduce.

1. (*3 points*) Draw the field line diagram on the reverse of this sheet for the electric field in between and to either side of the plates.
Shown on the reverse. Notice that there is no magnetic field outside the plates, and that it is uniform between the plates, pointing to the right. No edge effects because the plates are infinite. Additionally, from symmetry we know that the field if it exists must only point in the $\hat{x}$ direction, and could only possibly depend on x. So, the yellow Gaussian surface (Sorry its hard to see, but it encloses both plates), should convince you that the field outside the plates is zero. (Especially since the enclosed charge does not change, even as we move the outer boundaries in and out as we choose outside of the plates. The other surfaces should convince you that we have some field inside.

2. (*2 points*) What is the magnitude of the electric field between the plates? (Hint: use superposition and/or Gauss' Law)
So, since we know that the field outside is zero, we can just consider the green Gaussian surface in order to determine the field between the plates. We have, if the area of the cap (not visible in profile) is A, then Gauss's law tells us that (utilizing our symmetry arguments from the part above)

$$\int \vec{E} \cdot d\vec{A} = E(x)A = \frac{Q}{\epsilon_0} = \frac{\rho(Aw)}{\epsilon_0}$$

so we obtain our answer

$$\vec{E} = \frac{\rho w}{\epsilon_0}\hat{x}$$

so the magnitude

$$\boxed{E = \frac{\rho w}{\epsilon_0}}$$

3. (*1 point*) What is the magnitude of the electric field to the left of plates?
Its zero for the reasons we listed above

$$\boxed{E = 0}$$

4. (*1 point*) What is the magnitude of the electric field to the right of the plates?
Its zero for the reasons we listed above

$$\boxed{E = 0}$$

5. (*2 points*) What is the electric field inside the plate on the left? Be sure to explain any coordinates you introduce.

Alright, now we have to consider a Gaussian surface that starts left of the left plate, and terminates some distance w_1 into the plate, where I have marked w_1 on my diagram, it starts at the left edge of the plate, and takes the value w at the right edge of the plate.

First, without doing Gauss' Law we know that the field is zero at the left end of the plate, and $\rho w/\epsilon_0$ at the right edge of the plate and ought to grow linearly in between, the answer is

$$\boxed{\vec{E}(w_1) = \frac{\rho w_1}{\epsilon_0}}\hat{x}$$

Which grows linearly and has the proper boundary conditions. But, we can demonstrate the same fact with a Gaussian integral

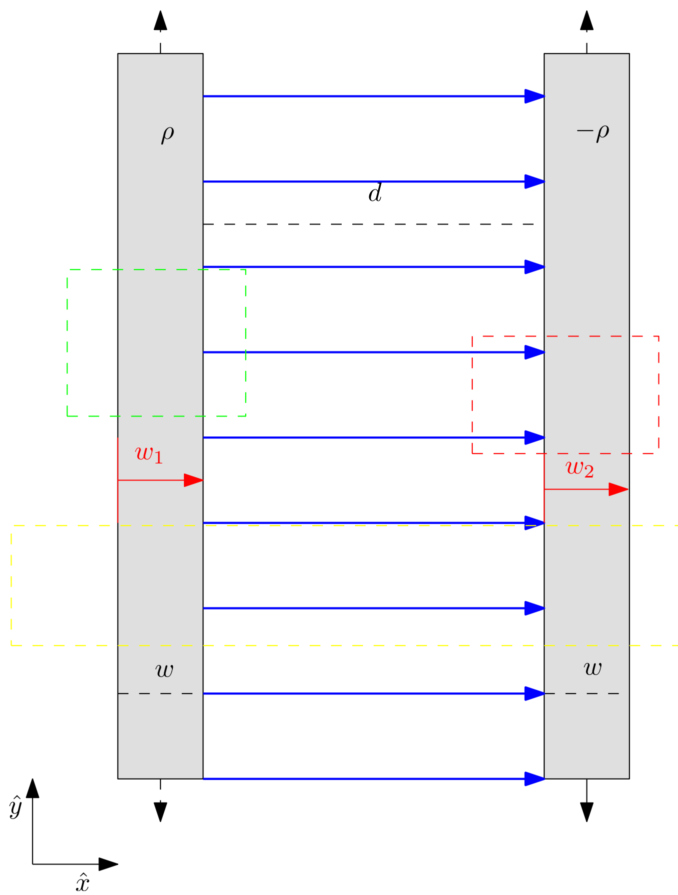
$$\int \vec{E}\cdot d\vec{A} = E(w_1)A = \frac{Q}{\epsilon_0} = \frac{\rho(Aw_1)}{\epsilon_0}$$

which yields the same result.

6. (*1 point*) What is the electric field inside the plate on the right? Be sure to explain any coordinates you introduce.

The answer ought to be the same in character as the previous question, but the details are a little different. If we define the coordinate w_2 as indicated in the diagram, then this time we want the field at $w_2 = 0$ to be $\rho w/\epsilon_0$ and at $w_2 = w$ to be zero, The solution is

$$\boxed{\vec{E}(w_2) = \frac{\rho}{\epsilon_0}(w - w_2)\hat{x}}$$



ρ
$-\rho$
d
w_1
w_2
w
w
$\hat{y}$
$\hat{x}$