2213 - Heat and E&M - 2009
Quiz - 09/16/09 Name: ______________________________ §#:

Show all work clearly and in order, and box your final answers. Simplify your expressions as best you can. Use the back of the sheet if you need to. You have 10 minutes to take this quiz.

POSITIVE AND NEGATIVE CHARGE - Two charges of magnitude q, are placed on the x-axis. The positive one at $x = a$, and the negative one at $x = -a$.

1. (*2 points*) What is the magnitude of the electric field at the origin?
We can sum the contributions from the two charges, We have

$$\vec{E} = -\frac{kq}{a^2}\hat{x} + -\frac{kq}{a^2}\hat{x} = -\frac{2kq}{a^2}$$

or in magnitude

$$\boxed{E = \frac{2kq}{a^2}}$$

2. (*1 point*) In what direction does it point?
To the left, or $\boxed{-\hat{x}}$.

3. (*3 points*) What is the magnitude of the electric field along the y axis?
Here again we need to add the fields. The resultant field will point in the $-\hat{y}$ direction. So we need to add only the y components of the fields.

$$\vec{E}(y) = \frac{kq(-\hat{y})}{a^2 + y^2}\frac{a}{\sqrt{a^2 + y^2}} + \frac{k(-q)\hat{y}}{a^2 + y^2}\frac{a}{\sqrt{a^2 + y^2}}$$

Simplifying we obtain

$$\vec{E}(y) = -\frac{2kqa}{(a^2 + y^2)^{3/2}}\ \hat{y}$$

or in magnitude

$$\boxed{E(y) = -\frac{2k(qa)}{(a^2 + y^2)^{3/2}}}$$

4. (*1 point*) In what direction does it point?
To the left, i.e. $\boxed{-\hat{x}}$

5. (*3 points*) How does the electric field on the y axis depend on y for $y \gg a$? Does this make sense?
We start with our result from above:

$$\vec{E}(y) = -\frac{2kq}{(a^2 + y^2)^{3/2}}\ \hat{y}$$

and write it in a form amenable to expansion

$$\vec{E}(y) = -\frac{2kqa}{y^3\left(1 + \left(\frac{y}{a}\right)^2\right)^{3/2}}\hat{y}$$

which we expand

$$\vec{E}(y) = -\frac{2kqa}{y^3}\left(1 + \frac{3}{2}\left(\frac{y}{a}\right)^2\right)\hat{y}$$

so to first order we obtain

$$\vec{E}(y) = -\frac{2k(qa)}{y^3}\hat{y}$$

or

$$\boxed{E(y) \propto 1/y^3}$$

This makes sense, it looks alot like the field due to a dipole. It decays faster than $1/y^2$ as we would expect for an isolated charge.