2213 - Heat and E&M - 2009
Quiz - 09/09/09 Name: ______________________________

Show all work clearly and in order, and box your final answers. Simplify your expressions as best you can. Use the back of the sheet if you need to. You have 10 minutes to take this quiz..

GAS IN A CYLINDER - A vertical metal cylinder of radius R is filled with ideal gas and covered with a piston, as shown in the diagram. The piston is free to move in the vertical direction. Initially, a block of mass m is resting on top of the piston, and the volume of the gas is V_0. The cylinder is in thermal equilibrium with its surrounding. (You may assume that the mass of the piston is negligible. You may use the value for the atmospheric pressure, P_{atm})

1. (*4 points*) What is the volume of the gas (V_f) after the block is removed and the cylinder is allowed to sit there for a while? (Simplify your expression as much as you can)

$$P_1V_1 = P_2V_2$$
$$V_2 = \frac{P_1}{P_2}V_1$$
$$V_f = \frac{P_1}{P_{\text{atm}}}V_0$$

and we know that

$$P = \frac{F}{A} = \frac{mg}{\pi R^2}$$

so we obtain

$$P_1 = P_{\text{atm}} + \frac{mg}{\pi R^2}$$

with the answer

$$V_f = \frac{P_{\text{atm}} + \frac{mg}{\pi R^2}}{P_{\text{atm}}}V_0$$
$$\boxed{V_f = \left(1 + \frac{mg}{P_{\text{atm}}\pi R^2}\right)V_0}$$

2. (*4 points*) What happens in the limit as $m \to 0$? Does this make sense?
When we take the limit we find that

$$V_f = \left(1 + \frac{mg}{P_{\text{atm}}\pi R^2}\right)V_0$$
$$V_f = V_0$$

This makes sense, if we remove the mass from the problem, we are asking if we have a cylinder filled with a gas at a certain temperature, and it has some initial volume V_0, after we do nothing, whats the volume? Its $V_f = V_0$.

3. (*4 points*) What happens in the limit as $m \to \infty$? Does this make sense?
If we take the limit $m \to \infty$ we discover

$$V_f = (1 + \infty)V_0 = \infty$$

I.e. our final volume should also approach infinity. This kind of makes sense, in that if we are weighing the gas down a ton before, after we remove that mass, we expect the gas to expand a lot. However, there is a problem in that our equation really isn't meant to handle this limit intelligently. Long before the mass becomes infinite, our Ideal Gas Law will no longer be applicable, shortly after that, the gas will condense, and a while after that it will solidfy, and a while after that quantum effects will become important, and then a while after that it will become a black hole, and then a while after that no one really knows what would happen. So, we learn a useful lesson: Our equations in physics have a realm of applicability, and we will get goofy results if we ask them what happens outside those realms, but the qualitative result (that the volume should increase a lot) makes sense, the infinity doesn't.